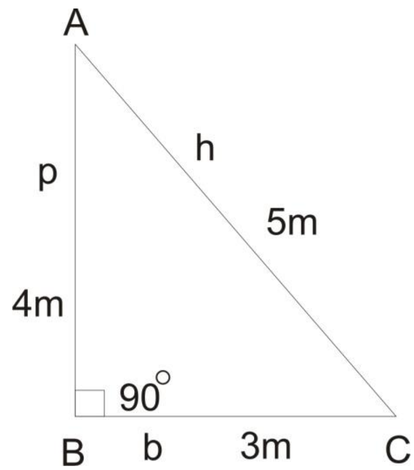
MENSURATION

Definition

1. **Mensuration :** It is a branch of mathematics which deals with the lengths of lines, areas of surfaces and volumes of solids.
2. **Plane Mensuration :** It deals with the sides, perimeters and areas of plane figures of different shapes.
3. **Solid Mensuration :** It deals with the areas and volumes of solid objects.

Important Formulae

Right Angled Triangle :



$(AC)^2 = (AB)^2 + (BC)^2$

or, $h^2 = p^2 + b^2$

If AC = 5m, AB = 4m then

$(BC)^2 = (AC)^2 - (AB)^2$

$= 25 – 16 = 9$

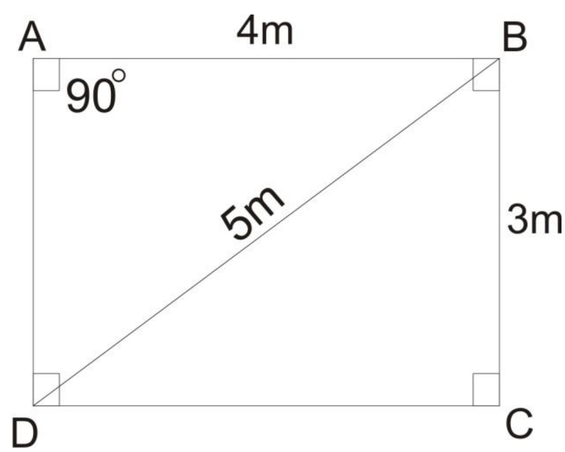
$\therefore$ BC = 3m

Rectangle : A rectangle is a plane,

Whose opposite sides are equal and

diagonals are equal. Each angle is

equal to 90°.



Here AB = CD; length l = 4m

AD = BC; breadth b = 3m

1. Perimeter of a rectangle = 2(length + breadth)

= 2(l + b)

= 2(4 + 3) = 14 m

2. Area of rectangle = length x breadth = l x b = 4 x 3

$= 12\ m^2$

3. Length of a rectangle : $\frac{area}{breadth} = \frac{A}{b} = \frac{12}{3}$ = 4 m

or, [$\frac{perimeter}{2}$ - breadth] = ($\frac{14}{2}$ - 3) = 4 m

Breadth of a rectangle : $\frac{area}{length} = \frac{A}{l} = \frac{12}{4}$ = 3 m

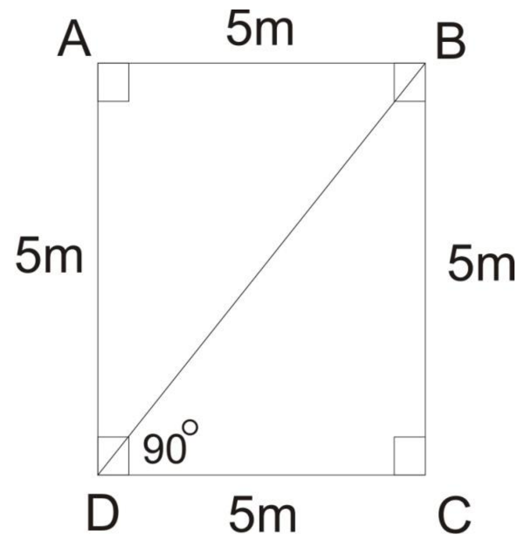
or, [$\frac{perimeter}{2}$ - length] = ($\frac{14}{2}$ - 4) = 3 m

4. Diagonal of rectangle : $\sqrt{(length)^2 + (breadth)^2}$

$= \sqrt{l^2 + b^2} = \sqrt{4^2 + 3^2}$

$= \sqrt{16 + 9} = \sqrt{25}$ = 5 m

Square : A square is a plane figure Bounded by four equal sides having all its angle as right angles.



Here AB = BC = CD = AD = 5 m = a(Let)

1. Perimeter of square = 4 x sides = 4a

= 4 x 5 = 20m

2. Area of a square = $(sides)^2 = a^2 = (5)^2$ = 25 sq. m

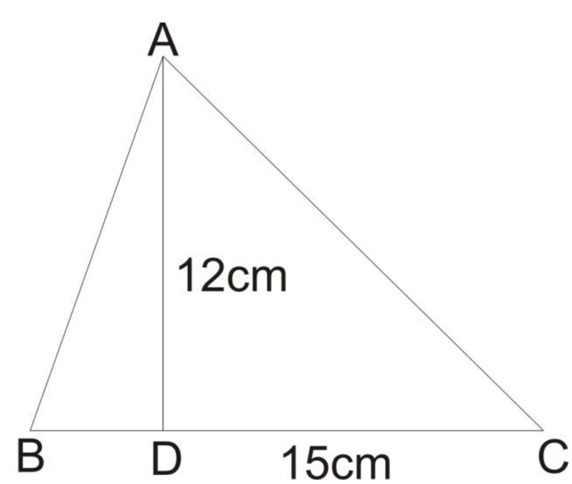
3. Side of a square = $\sqrt{area} = \sqrt{25}$ = 5m or, $\frac{Perimeter}{4} = \frac{20}{4}$ = 5 m

4. Diagonal of a square = $\sqrt{2}$ x side = $\sqrt{2}$ a

= $\sqrt{2}$ x 5 = $5\sqrt{2}$ m

5. Side of a square = $\frac{diagonal}{\sqrt{2}} = \frac{5\sqrt{2}}{\sqrt{2}}$ = 5m

Triangle :



1. Area of triangle = $\frac{1}{2}$ x base x height = $\frac{1}{2}$ x b x h

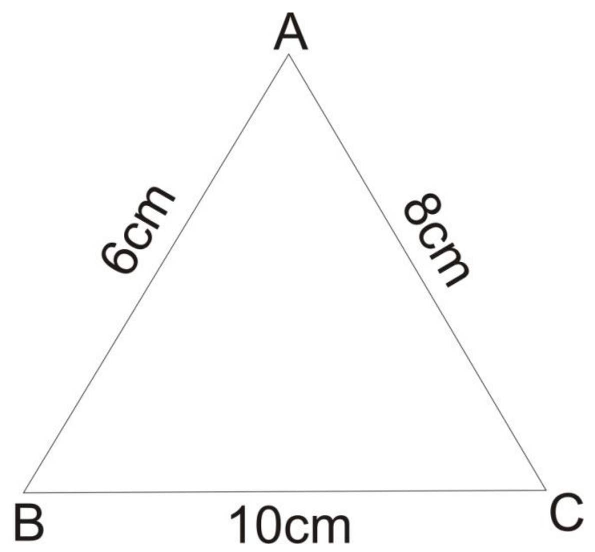
= $\frac{1}{2}$ x 15 x 12 = 90 sq. cm

here AD = 12 cm = height, BC = 15 cm = base

2. Semi perimeter of a triangle

$$S = \frac{a+b+c}{2} = \frac{10+8+6}{2} = 12 \text{ cm}$$

here BC = a, AC = b, AB = c



3. Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

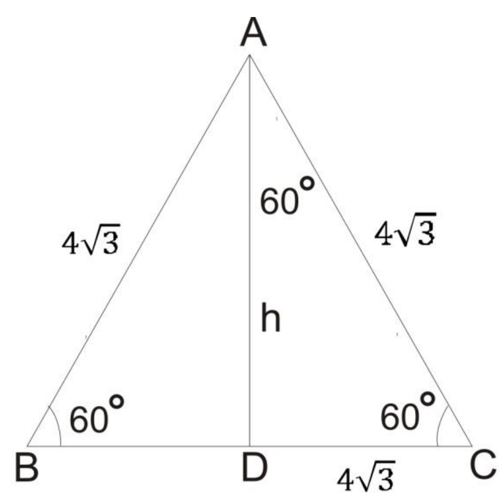
where a = 10cm, b = 8cm, c= 6cm, s= 12cm

= $\sqrt{12(12-10)(12-8)(12-6)}$

= $\sqrt{12 \times 2 \times 4 \times 6}$ = 24 cm^2

4. Perimeter of a triangle = 2s = (a + b + c)

= 10 + 8 + 6 = 24 cm



5. Area of an equilateral triangle = $\frac{\sqrt{3}}{4}$ x $(\text{side})^2$

= $\frac{\sqrt{3}}{4}$ x $(4\sqrt{3})^2$

= $\frac{\sqrt{3}}{4}$ x 48 = $12\sqrt{3}$ cm^2

6. Height of an equilateral triangle = $\frac{\sqrt{3}}{2}$ x $(\text{side})^2$ = $\frac{\sqrt{3}}{2}$ x $4\sqrt{3}$

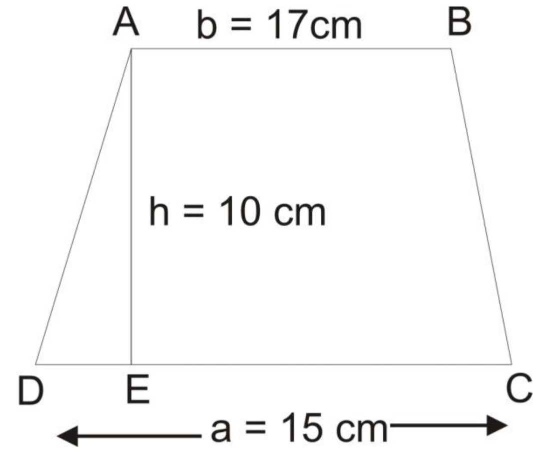
= 6 cm

7. Perimeter of an equilateral triangle = 3 x (side)

= 3 x $4\sqrt{3}$ = $12\sqrt{3}$ cm

Quadrilateral :

Parallelogram :



(i) Area of parallelogram = base x height

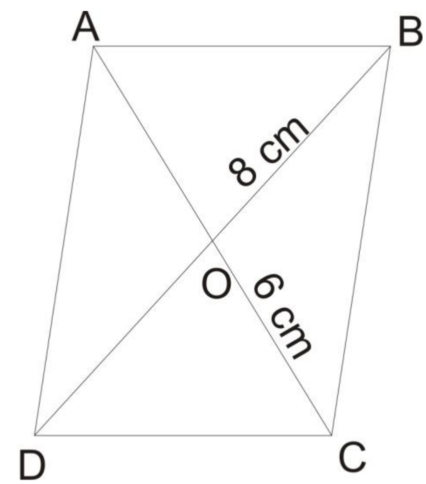
= b x h

= 8 x 5 = 40 sq.cm.

(ii) Perimeter of a parallelogram = 2(AB + BC)

$= 2(8 + 5) = 26$ cm

Rhombus :



(i) Area of rhombus = $\frac{1}{2}$ x (product of diagonals)

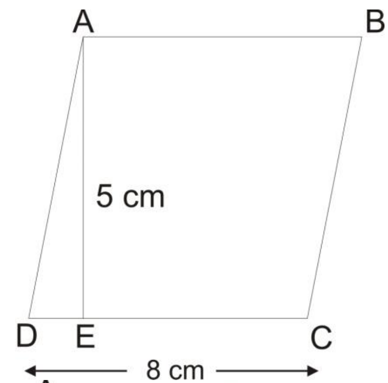
$$= \frac{1}{2}(d_1 \cdot d_2) = \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$$

(ii) Perimeter of rhombus = 4 x side = 4a

here AB = BC = CD = AD = 4a

AC = d_1, BD = d_2

Trapezium :



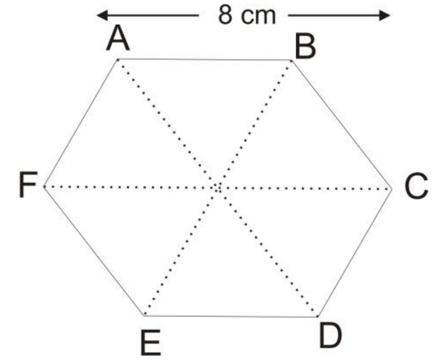
(i) Area if a trapezium = $\frac{1}{2}$ x (sum of parallel sides) x height

$$= \frac{1}{2} \times (a + b) \times h$$

$$= \frac{1}{2} \times (15 + 17) \times 10$$

$$= \frac{1}{2} \times 32 \times 10 = 160 \text{ cm}^2$$

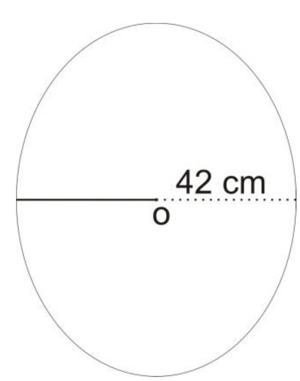
Regular Hexagon :



(i) Area of a regular hexagon = $6 \times \frac{\sqrt{3}}{4} \times (\text{side})^2$

(ii) Perimeter of a regular hexagon = 6 x (side)

Circle :



(i) Circumference of a circle = π x diameter

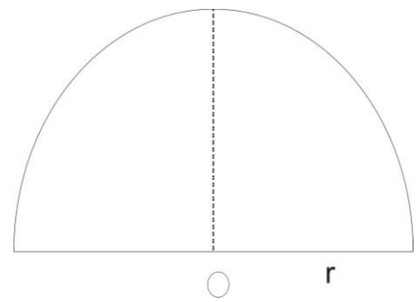
$$= \pi \times 2r = 2\pi r$$

$$= 2 \times \frac{22}{7} \times 42 = 264 \text{ cm}$$

(ii) Radius of a circle = $\frac{circumference}{2\pi} = \frac{264 \times 7}{2 \times 22}$ = 42 cm

(iii) Area of a circle = $\pi \times r^2 = \frac{22}{7} \times 42^2 = \frac{22}{7} \times 42 \times 42 = 5544 \text{ cm}^2$

(iv) Radius of a circle = $\sqrt{\frac{area}{\pi}}$



$$= \sqrt{\frac{5544}{22} \times 7} = \sqrt{1764} = 42 \text{ cm}$$

(v) Area of a semi circle = $\frac{1}{2}\pi r^2 = \frac{1}{8}\pi d^2$

$$= \frac{1}{2} \times \frac{22}{7} \times 42^2 = 2772 \text{ cm}^2$$

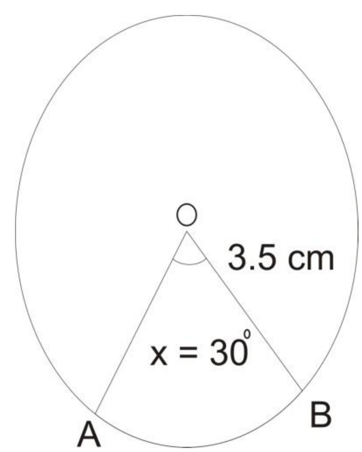
(vi) Circumference of semi circle = $\frac{22}{7} \times 42 = 132$ cm

(vii) Perimeter of semi circle = $(\pi r + 2r) = (\pi + 2)\, r = (\pi + 2)\, \frac{d}{2}$

(viii) Area of sector OAB = $\frac{x}{360} \times \pi r^2$

(x being the central angle)

$= \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 3.5 \times 3.5 = 3.21$ sq. m.



(ix) Central angle by arc AB = $360^\circ \times \frac{area\ of\ OAB}{area\ of\ circle}$

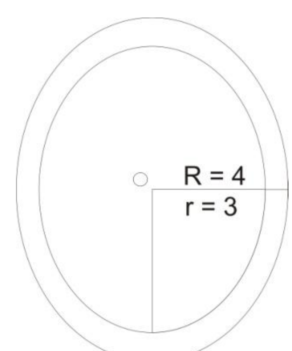
$$= 360^\circ \times \frac{3.21}{\frac{22}{7} \times 3.5 \times 3.5} = \frac{360 \times 321}{22 \times 35 \times 5} = 30^\circ \text{(approx)}$$

(x) Radius of circle = $\sqrt{\frac{360^\circ}{central\ angle\ by\ arc} \times \frac{area\ of\ OAB}{\pi}}$

$$= \sqrt{\frac{360^\circ}{30^\circ} \times \frac{3.21}{\frac{22}{7}}} = \sqrt{\frac{134.82}{11}} = \sqrt{12.23} = 3.5 \text{ m.}$$

(xi) Area of ring

= difference of the area of two circle



$= \pi R^2 - \pi r^2 = (R^2 - r^2)$

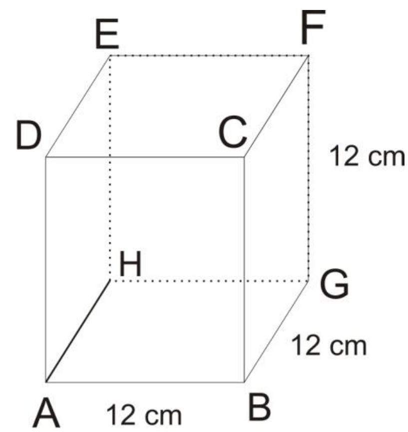
$= \pi(R + r)(R - r)$

= (sum of radius)(diff. of radius)

$= \frac{22}{7} \times (4 + 3)(4 - 3) = \frac{22}{7} \times 7 \times 1$

= 22 sq. cm.

Cuboid and Cube :



(i) Total surface area of cuboid

= 2(lb + bh + hl) sq. unit

Here l = length, b = breadth, h = height

= 2(12 × 8 + 8 × 6 + 6 × 12)

= 2(96 + 48 + 72) = 2 × 216 = 432 sq. cm.

(ii) Volume of a cuboid = (length × breadth × height) = lbh

= 12 × 8 × 6 = 576 cuboic cm

(iii) Diagonal of a cuboid $= \sqrt{l^2 + b^2 + h^2} = \sqrt{12^2 + 8^2 + 6^2}$

$= \sqrt{144 + 64 + 36} = \sqrt{244} = 2\sqrt{61}$ cm.

(iv) Length of cuboid $= \frac{Volume}{Breadth \times Height} = \frac{v}{b \times h}$

(v) Breadth of cuboid $= \frac{Volume}{Length \times Height} = \frac{v}{l \times h}$

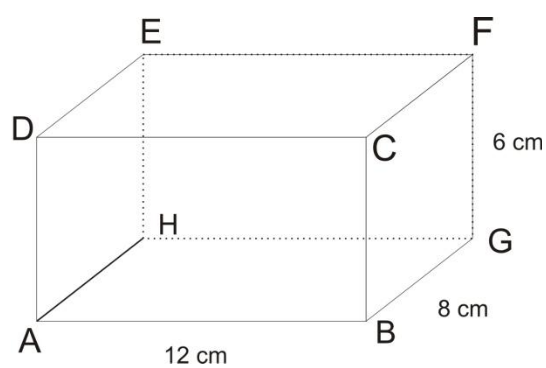
(vi) Height of cuboid $= \frac{Volume}{Length \times Breadth} = \frac{v}{l \times b}$

(vii) Volume of cube = $(\text{side})^3$

$= 12^3$

= 1728 cubic cm

Cube : All sides are equal = 12 cm



(viii) Sides of a cube $= \sqrt[3]{Volume}$

$= \sqrt[3]{1728} = 12$ cm

(ix) Diagonal of cube = $\sqrt{3} \times$ (side) = $\sqrt{3} \times 12 = 12\sqrt{3}$ cm

(x) Total surface area of a cube = 6 × $(\text{side})^2$ = 6 × 12^2 = 864 sq.cm

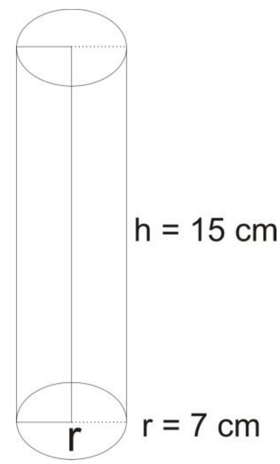
Right Circular Cylinder :

(i) Area of curved surface

= (perimeter of base) x height

= $2\pi rh$ sq. unit

= $2 \times \frac{22}{7} \times 7 \times 15 = 660$ sq. cm



(ii) Total surface area = area of circular ends + curved surface area

= $2\pi r^2 + 2\pi rh = 2\pi r(r + h)$ sq. unit

= $2 \times \frac{22}{7} \times 7(15 + 7)$

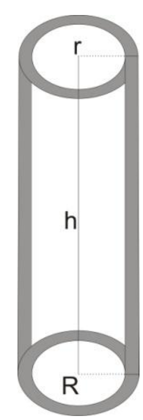
= 2 x 22 x 22

= 968 sq. cm.

(iii) Volume = (area of base) x height

= $(\pi r^2) \times h = \pi r^2 h$

= $\frac{22}{7} \times 7 \times 7 \times 15 = 2310$ cubic cm.



(iv) Volume of a hollow cylinder = $\pi R^2 h - \pi r^2 h$

= $\pi h(R^2 - r^2) = \pi h\,(R + r)(R - r)$

= π x height x (sum of radii)(difference of radii)

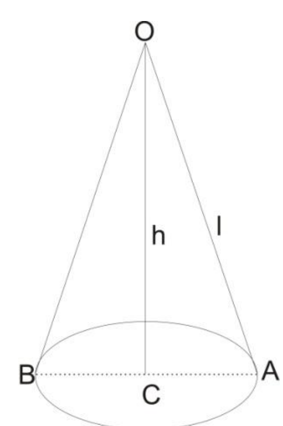
Here R, r are outer and inner radii respectively and h is the height.

Cone :

(i) In right angled ΔOAC, we have

$l^2 = h^2 + r^2$

(here r = 35 cm, l = 37 cm, h = 12 cm)



Or, $l = \sqrt{h^2 + r^2}$

$h = \sqrt{l^2 - r^2}$, $r = \sqrt{l^2 - h^2}$

where l = slant height, h = height, r = radius of base

(ii) Curved surface area = $\frac{1}{2}$ x (perimeter of base) x slant height

$= \frac{1}{2} \times 2\pi r \times l = \pi rl$ sq. unit

$= \frac{22}{7} \times 35 \times 37 = 4070$ sq. cm

(iii) Total surface area S = area of circular base + curved surface area

$= (\pi r^2 + \pi rl) = \pi r(r + l)$ sq. unit

$= \frac{22}{7} \times 35(37 + 35) = 7920$ sq. cm

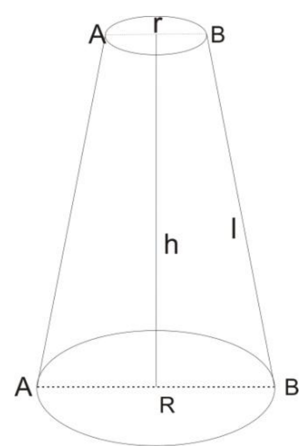
(iv) Volume of cone = $\frac{1}{3}$ (area of base) x height

$= \frac{1}{3}(\pi r^2) \times h = \frac{1}{3}\pi r^2 h$ cubic unit

$= \frac{1}{3} \times \frac{22}{7} \times 35 \times 35 \times 12$

= 15400 cubic cm

Frustum of Cone :



(v) Volume of frustum = $\frac{1}{3}\pi h(R^2 + r^2 + Rr)$ cubic unit

(vi) Lateral surface = $\pi l(R + r)$

where $l^2 = h^2 + (R - r)^2$

(vii) Total surface area = $\pi[R^2 + r^2 + l(R + r)]$

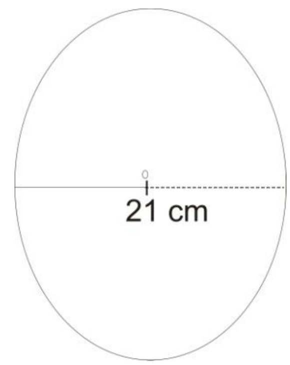
R, r be the radius of base and top the frustum ABB'A' h and l be the vertical height and slant height respectively.

Sphere :

(i) Surface area = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times (10.5)^2 = 1386 \text{ sq. cm}$$

here, d = 21 cm $\therefore$ r = 10.5 cm

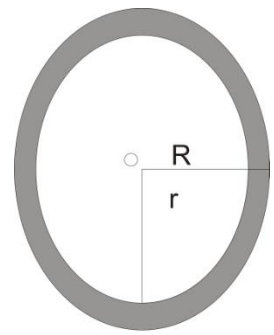


(ii) Radius of sphere = $\sqrt{\frac{Surface\ area}{4\pi}} = \sqrt{\frac{1386 \times 7}{4 \times 22}}$ = 10.5 cm

(iii) Diameter of sphere = $\sqrt{\frac{Surface}{4\pi}} = \sqrt{\frac{1386 \times 7}{22}}$ = 21 cm

(iv) Volume of sphere V = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(\frac{d}{2})^3 = \frac{1}{6}\pi d^3$

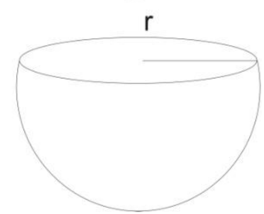
$$= \frac{1}{6} \times \frac{22}{7} \times 21 \times 21 \times 21 = 4831 \text{ cubic cm}$$



(v) Radius of sphere = $\sqrt{\frac{3}{4\pi}\ x\ Volume\ of\ sphere}$

(vi) Diameter = $\sqrt[3]{\frac{6 \times V}{\pi}}$

(vii) Volume of spherical ring = $\frac{4}{3}\pi(R^3 - r^3)$



(viii) Curved surface of hemisphere = $2\pi r^2$

(ix) Volume of hemisphere = $\frac{2}{3}\pi r^3$

(x) Total surface area of hemisphere = $3\pi r^2$

Note : V = volume, A = area, h = height, b = base,breadth, d= diameter, R = outer radius, r = inner radius, $\pi = \frac{22}{7}$ = 3.142, a = side.

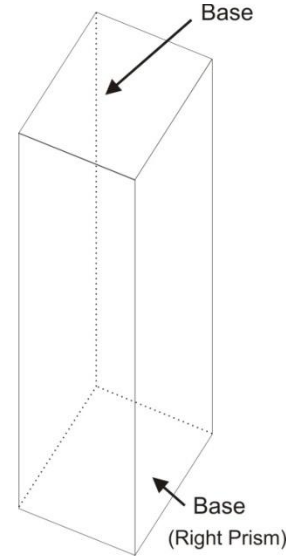
Prism and Pyramid

Prism

1. **Solid :** Bodies which have three dimensions in space are called solid. For example, a block of wood.

   A body, which has the three dimensions length, breadth and height, is a solid, whereas a rectangle with its two dimensions (length and breadth) is not a solid.

2. **Prism :** A prism is a solid, bounded by plane faces of which two opposite sides known as bases are parallel and congruent polygons.
3. **Base :** The congruent and parallel faces of a



prism are called its bases.

The other faces of a prism can be either oblique to the faces or perpendicular to them.

4. **Right prism :** A right prism is a prism in which lateral sides are rectangular or perpendicular to their bases.
5. **Lateral faces :** The side faces of a prism are called its lateral faces.
6. **Lateral surface area :** The area of all the lateral faces of a prism is called its lateral surface area.

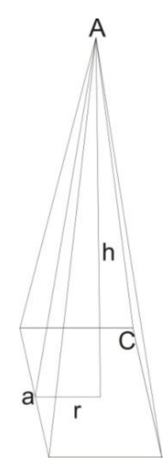
Note : In a right prism having polygons of n sides as bases.

(i) the number of vertices = 2
(ii) the number of edges = 3n
(iii) the number of lateral faces = (n + 1), and
(iv) all the lateral faces are rectangular.

Formulae

(i)	Volume of a right prism = (Area of its base) x height
(ii)	Lateral surface area of a right prism =(perimeter of its base) x height
(iii)	Total surface area of a right prism =(lateral surface area) + 2(area of the base)

Pyramid



1. **Pyramid :** A solid of triangular lateral sides having a common vertex and plane rectilinear bases with equal sides is called pyramid.
2. **Height of the pyramid :** The length

of perpendicular drawn from the vertex

of a pyramid to its base is called the

height of the pyramid.

The side faces of pyramid form its lateral surface.

3. **Regular pyramid :** If the base of a pyramid is a regular figure i.e., a polygon with all sides equal and all angles equal, then it is called a regular pyramid.
4. **Right pyramid :** If the foot of the perpendicular from the vertex of a pyramid to its base is the centre of the base then it is called a right pyramid.
5. **Slant height of a regular right pyramid :** The slant height of a regular right pyramid is the length of the line segment joining the vertex to the mid-point of one of the sides of the base.
6. **Tetrahedron :** When the base of a right pyramid is a triangle, then it is called a tetrahedron.
7. **Regular tetrahedron :** A right pyramid with equilateral triangle as its base is called a regular tetrahedron.