Differential Equation

Many physical laws & relation between many physical quantity mathematically express in term of differential equation.

Definition :- "An equation containing dependent & independent variable, differential coefficient of dependent variable of several order is called as differential equation".

For example:- In Newton's Law of Cooling, if T= Body temperature, T= Surrounding temperature then, by law, rate of change of body temperature ($\frac{dT}{dt}$) is directly proportional to difference between body temp. & surrounding temperature ($T - T_O$).

i.e. Mathematically, $\frac{dT}{dt} \sim (T - T_0)$

or $\frac{dT}{dt} = -k(T - T_0)$. (-ve sign for cooling process)

This is first order differential equation

Order of Differential Equation:- " The order of highest derivative occurs in given equation is called as order of differential equation".

For example:-

1) $\frac{dy}{dx} + 3x + y = 0$, Order = 1
2) $\left(\frac{d^2y}{dx^2}\right)^3 + 3\frac{dy}{dx} - 2y = 0$, Order = 2

Degree of Differential equation:-" The power of highest derivative occurs in given equation when all term contain differential coefficient cleared from radical sign or fractional power ".

For example:-

1) $\left(\frac{d^2y}{dx^2}\right)^3 + 3\frac{dy}{dx} - 2y = 0$, Degree = 3
2) $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + 9y = 0$, Here power of highest derivative is 1, so Degree = 1
3) $\frac{d^2y}{dx^2} = \sqrt{1 + \frac{dy}{dx}}$, In this D. E. power of highest derivative is 1 but there is fraction power (root sign) in equation so degree of equation is not 1. For Degree we have to express fraction power into integer power.

So, by squaring on both side,

$$\left(\frac{d^2y}{dx^2}\right)^2 = \left(\sqrt{1+\frac{dy}{dx}}\right)^2$$

Or , $\left(\frac{d^2y}{dx^2}\right)^2 = 1+\frac{dy}{dx}$. Now power of highest derivative is 2.

So, **Degree** of equation is **2**.

Assignment 1

Q. Find Order & Degree Of following D. E.

1) $1+\left(\frac{dy}{dx}\right)^2 + \frac{d^2y}{dx^2} = 0$

Solution :- Order = 2

Since power of highest derivative is 1. So, **Degree = 1**

2) $\left(\frac{dy}{dx}\right)^3 + 3y = \left(\frac{d^2y}{dx^2}\right)^2$

Solution :- Order = 2

Degree =

3) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$

Solution :-

4) $\sqrt[3]{\frac{d^2y}{dx^2}} = \sqrt{\frac{dy}{dx}}$

Solution :-

5) $$\frac{y - x\frac{dy}{dx}}{\frac{d^2y}{dx^2}} = \left(\frac{d^2y}{dx^2}\right)^2$$

Solution :-

Exercise- 1

Find Order & Degree Of following D. E.

1) $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + xy = 0$

3) $\sqrt{\frac{d^2y}{dx^2}} - \frac{dy}{dx} - xy^2 = 0$

2) $\left(\frac{d^3y}{dx^3}\right)^{\frac{1}{2}}\left(\frac{dy}{dx}\right)^{\frac{1}{3}} = 3$

4) $\left[1 + \left(\frac{dy}{dx}\right)^3\right]^{\frac{5}{3}} = 2\frac{d^2y}{dx^2}$

<u>Solution of differential Equation:-</u> “ A relation between dependent and independent variable which do not contain any differential coefficient, satisfies given differential equation is called as Solution of D. E.”

<u>General Solution :-</u> “ A solution which contain number of arbitrary constant equal to order of differential equation is called as General solution”.

$\frac{dy}{dx} = \cos x$ ► **Solution:- y = sinx,** ► **General solution:- y = sinx + C,**

(C= constant)

<u>Formation of Diff. Equation:-</u> First count the No. of constant in given solution so that order of corresponding Diff. Equation is known. That No. of time differentiate given relation & eliminate all constant from the equation. Corresponding differential equation is obtained.
<u>For Example:-</u>

Let, y =A cosx + B sinx ------------(1)

There are 2 arbitrary constant i.e. A, B.

Diff. eq. (1) w.r.t.x

$$\frac{dy}{dx} = -A\sin x + B\cos x$$

Diff. w.r.t.x again,

$$\frac{d^2y}{dx^2} = -A\cos x - B\sin x$$

$$= -(A\cos x + B\sin x)$$

From eq. (1),

$$\frac{d^2y}{dx^2} = -y \quad ► \quad \textbf{D.E. is } \frac{d^2y}{dx^2} + y = 0$$

<u>Assignment-2</u>

Q.1) Form differential equation from solution, y = A cos(logx) + B sin(logx).

<u>Solution:-</u> Let, y = A cos(logx) + B sin(logx) ------------------(1)

There are two constant A & B.

Diff. w.r.t.x,

$$\frac{dy}{dx} = -A\sin(\log x).\frac{1}{x} + B\cos(\log x).\frac{1}{x}$$

Multiply by 'x' on both side

$$x\frac{dy}{dx} = -A\sin(\log x) + B\cos(\log x)$$

Diff. w.r.t.x

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}=-A\cos(\log x)\frac{1}{x}-B\sin(\log x)\frac{1}{x}$$

D.E. is ---

2) $y=Ae^{2x}+Be^{-2x}$

Solution:- Let, $y=Ae^{2x}+Be^{-2x}$

Diff. w.r.t.x,

$$\frac{dy}{dx}=2Ae^{2x}-2Be^{-2x}$$

Again Diff. w.r.t.x,

D.E. is ---

3) $y=ae^{-9x}\cos(3x+b)$ **,** where a and b are arbitrary constant

Solution:- Let, $y=ae^{-9x}\cos(3x+b)$

D.E. is ---

4) $y = mx + c$, m and c are constant

Solution:-

D.E. is ---

5. $y = A\cos 4x + B\sin 4x + C$

Solution:-

D.E. is --

Exercise:-2

Form a differential equation corresponding to following equation,

1) $y = ae^x$
2) $y = Ax + \frac{B}{x}$
3) $Ax^2 + By^2 = 1$
4) $y = e^x(A\cos x + B\sin x)$

► **Solution of First order, First Degree Differential Equation**

Every first order, first degree diff. equation is generally expressed in the form M dx + N dy = 0, where M & N are function of x & y or constant term. Depending On nature of function M & N, there are different types of diff. equation.

Types of Differential equation:-

1) Variable Separable Form
2) Homogeneous diff. equation
3) Exact diff. equation
4) Linear diff. equation
5) Bernoulli's diff. equation

1)Variable Separable form:- If differential equation can be expressed in such a way that all x variable term are occur in one side & y variable term occur in other side of equation so that both variable term are separated on opposite side of equation, that form is called as variable separable form of diff. equation.

i.e. $f(x)dx = g(y)dy$

For obtaining solution of such form, integrate this form & **introduced one arbitrary Constant.**

i.e. $\int f(x)dx = \int g(y)dy + c$

For Example:- Solved : $\frac{dy}{dx} = \frac{y}{x}$

By simplifying,

$$\frac{dy}{y} = \frac{dx}{x} \qquad \text{(Var.sep.form)}$$

Integrate,

$$\int \frac{dy}{y} = \int \frac{dx}{x} \qquad \text{------ (solution formula)}$$

logy = logx + C ------(Add arbitrary constant 'C')

logy – logx = logc (for convenience, assume C = logc)

$$\log\left(\frac{y}{x}\right) = \log c \quad \blacktriangleright \quad \frac{y}{x} = c \text{ or } \boxed{\mathbf{y = cx}} \text{ -------- (general solution)}$$

Assignment-3

Q. Solve following Diff. Equation

1) Solve : $(1 + x^3)dy - x^2 ydx = 0$

Solution:- $(1 + x^3)dy - x^2 ydx = 0$

By simplifying,

$$(1 + x^3)dy = x^2 ydx$$

$$\frac{dy}{y} = \frac{x^2 dx}{(1 + x^3)} \qquad \text{------------- (Variable Separable form)}$$

Integrate,

$$\int \frac{dy}{y} = \int \frac{x^2 dx}{(1 + x^3)}$$

General Solution is, -------------------------------------

2) Solve:- $\frac{dy}{dx}=e^{x-y}+xe^{-y}$

Solution:- Using law of indices, ($a^{m-n}=a^{m}a^{-n}$)

$$\frac{dy}{dx}=e^{x}e^{-y}+xe^{-y}$$

$$\frac{dy}{dx}=e^{-y}\left(e^{x}+x\right)$$

$$\frac{dy}{e^{-y}}=\left(e^{x}+x\right)dx$$ -------------- (Variable Separable form)

General Solution is, ---------------------------------------

3) Solve:- $y+x\frac{dy}{dx}=a\left(y^{2}+\frac{dy}{dx}\right)$

Solution:-

General Solution is, ---

4) Solve :- $(1+x)\frac{dy}{dx}+1=2e^{-y}$

Solution:-

General Solution is, ---

Exercise:-3

Solve : 1) $(x^2+1)dy-(y^2+1)dx=0$

2) $\frac{dy}{dx}=e^{3x-2y}+x^2e^{-2y}$

3) 2sinx siny dy = cosx cosydx

4) $(xy^2+x)dx+(yx^2+y)dy=0$

2) Homogeneous Differential Equation:-

Homogeneous Function:- A Function f(x,y) is called as homogeneous function if sum of power (indices) of x and y variable in every term of equation are equal.
The sum of power is called as Degree of homogeneous function.

Every homogeneous function can be expressed as $f(x,y)=x^n¤\left(\frac{y}{x}\right)$

An equation Mdx+Ndy = 0 is called as homogeneous diff. equation if M& N are homogeneous function of same degree.
For example:- $(x^2+y^2)dx-2xydy=0$ **--------- Homogeneous D. E.**
Solution of Homo. D. E.:- For solving homogeneous diff. equation,

Use substitution, $\boxed{\frac{y}{x}=t}$ ► i.e. $\boxed{y = x\,t}$ and $\boxed{\frac{dy}{dx}=x\frac{dt}{dx}+t}$

After this substitution diff. equation is reduced into variable separable form.

For Example:- Solve : $(x^2+y^2)dx-2xydy=0$

As $M=(x^2+y^2)$ & $N=-2xy$ are homogeneous function of degree 2.
It is homogeneous differential equation
By simplifying,

$$\frac{dy}{dx}=\frac{(x^2+y^2)}{2xy}$$ -------- (1) (Express equation in terms of $\frac{dy}{dx}$)

Put $\boxed{y = x\,t}$ and $\boxed{\frac{dy}{dx}=x\frac{dt}{dx}+t}$

Eq. (1) become,

$$x\frac{dt}{dx}+t=\frac{(x^2+x^2t^2)}{2x(xt)}$$

$$=\frac{x^2(1+t^2)}{x^2(2t)}$$

$$x\frac{dt}{dx} = \frac{1+t^2}{2t} - t$$

$$x\frac{dt}{dx} = \frac{1-t^2}{2t}$$

$$\left(\frac{2t}{1-t^2}\right)dt = \frac{dx}{x}$$ ------------------ (Variable separable form)

Integrating,

$$\int\left(\frac{2t}{1-t^2}\right)dt = \int\frac{dx}{x}$$

$$-\log(1-t^2) = \log x + \log c$$ (logc is arbitrary constant)

Using back substitution $\boxed{t = \frac{y}{x}}$ & simplification

General solution is **,** $\boxed{\left(x^2 - y^2\right) = Cx}$ (where $C = c^{-1}$)

Assignment-4

1) Solve :- $\left(xy - x^2\right)\frac{dy}{dx} = y^2$

Solution:- $\left(xy - x^2\right)\frac{dy}{dx} = y^2$

$$\frac{dy}{dx} = \frac{y^2}{\left(xy - x^2\right)}$$

Put $\boxed{y = x\,t}$ and $\boxed{\frac{dy}{dx} = x\frac{dt}{dx} + t}$

$$x\frac{dt}{dx} + t = \frac{x^2t^2}{x^2(t-1)}$$

$$x\frac{dt}{dx} = \frac{t^2}{t-1} - t$$

$$x\frac{dt}{dx} = \frac{t}{t-1}$$

$$\left(\frac{t-1}{t}\right)dt = \frac{dx}{x}$$ ------------------ (Variable separable form)

General solution is --

2) Solve:- $\frac{dy}{dx} = \frac{y}{x} + \sin\left(\frac{y}{x}\right)$

Solution:- It is homogeneous differential equation

Put $y = x\,t$ and $\frac{dy}{dx} = x\frac{dt}{dx} + t$

Equation become,

$$x\frac{dt}{dx} + t = t + \sin t$$

$$x\frac{dt}{dx} = \sin t$$

$$\operatorname{cosec} t\, dt = \frac{dx}{x}$$ --------------------- (Variable separable form)

General solution is --

3) Solve:- $yx^2 dx = \left(x^3 - y^3\right)dy$

Solution:-

General solution is --

4) Solve:- $\left(x^2 + y^2\right)dx + 8xydy = 0$

Solution:-

General solution is --

Exercise:-4

Solve, 1) $2xydx + (y^2 - x^2)dy = 0$

3) $\frac{dy}{dx} = \frac{4x-3y}{3x-2y}$

2) $\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$

4) $\frac{dy}{dx} = \frac{x^2+y^2}{xy}$ given that y = 2 when x = 1.

3) Exact Differential Equation:- An equation M dx + N dy = 0 is called as Exact differential equation if there exist such function u(x,y) for which M dx + N dy = du (Total differential).

i.e. M dx + N dy = $\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy$

Comparing both side,

$$M = \frac{\partial u}{\partial x} \text{ and } N = \frac{\partial u}{\partial y}$$

Diff. M w.r.t.y partially, N w.r.t.x partially

$$\frac{\partial M}{\partial y} = \frac{\partial^2 u}{\partial y \partial x} \text{ and } \frac{\partial N}{\partial x} = \frac{\partial^2 u}{\partial x \partial y} \quad \text{---------(1)}$$

But, mixed partial derivative are commutative (equal).

i.e. $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

From (1), $\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$ ----------- (condition of exactness)

Note:- An equation M dx + N dy = 0 if function M & N satisfied condition of exactness.

i.e. $\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$

General Solution Formula :- $\int_{y=constant} Mdx + \int$ (Terms of N not containing 'x') dy = C

For Example:- Solve :- $(x+y-2)dx + (x-y+4)dy = 0$

Solution:- This is of the type M dx + N dy = 0 Where , M = $(x+y-2)$ & N = $(x-y+4)$

$$\frac{\partial M}{\partial y}=\frac{\partial}{\partial y}(x+y-2)$$ = 0 + 1 – 0 = 1 -------- (1)

$$\frac{\partial N}{\partial x}=\frac{\partial}{\partial x}(x-y+4)$$ = 1 – 0 + 0 = 1 --------- (2)

From (1) & (2), $\boxed{\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}}$ So, Given Equation is Exact D. E.

Gen. Solution is,

$$\int(x+y-2)dx+\int(-y+4)dy=C$$

$\boxed{\frac{x^2}{2}+xy-2x-\frac{y^2}{2}+4y=C}$ ------------------------Gen. Solution

Assignment-5

1) Solve:- $(3x^2+6xy^2)dx+(6x^2y+4y^2)dy=0$

Solution:- Let, $M=3x^2+6xy^2$ & $N=6x^2y+4y^2$

$$\frac{\partial M}{\partial y}=\frac{\partial}{\partial y}(3x^2+6xy^2)=0+6x(2y)=12xy$$ -------------- (1)

$$\frac{\partial N}{\partial x}=\frac{\partial}{\partial x}(6x^2y+4y^2)=6y(2x)+0=12xy$$ -------------- (2)

From (1) & (2) , Equation is Exact.

Gen. Solution is ,

$$\int(3x^2+6xy^2)dx+\int(4y^2)dy=C$$

General solution is ---

2) Solve :- $(2xy+y^2)dx+(x^2+2xy+\sin y)dy=0$

Solution:- **:-** Let, $M=(2xy+y^2)$ & $N=(x^2+2xy+\sin y)$

$$\frac{\partial M}{\partial y}=\frac{\partial}{\partial y}(2xy+y^2)=2x+2y$$ ------------------------ (1)

$$\frac{\partial N}{\partial x}=\frac{\partial}{\partial x}(x^2+2xy+\sin y)=2x+2y$$ ---------------- (2)

From (1) & (2) , Equation is Exact.

General solution is ---

3) Solve :- $(2xy + y - \tan y)dx + (x^2 - x\tan^2 y + \sec^2 y)dy = 0$

Solution :-

General solution is --

4) Solve :- $\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$

Solution :-

General solution is ---

Exercise:-5

Solve:- 1) $(1+xy^2)dx+(1+x^2y)dy=0$ 2) $(2xy^4+\sin y)dx+(4x^2y^3+x\cos y)dy=0$

3) $(1+\log xy)dx+\left(1+\frac{x}{y}\right)dy=0$ 4) $\frac{dy}{dx}+\frac{y\cos x+\sin y+y}{\sin x+x\cos y+x}=0$

5) $(\sin x\cos y+e^{2x})dx+(\cos x\sin y+\tan y)dy=0$ 6) $\frac{dy}{dx}=\frac{5-3x-2y}{2x+3y-5}$

4) Linear Diff. Equation :- An Equation which expressed in the form $\boxed{\frac{dy}{dx}+P(x)y=Q(x)}$ is called as **'Linear Diff. Equation in y',** Where P(x) & Q(x) are function of x only or Constant.

Note:- 1) Any Equation is linear in y if it contain only one free term of y with first power.

2) In Linear Diff. Equation Coefficient of $\frac{dy}{dx}$ must be equal to **one**.

Method of Solution:- After expressing linear equation in the form $\boxed{\frac{dy}{dx}+P(x)y=Q(x)}$

1) First find integrating factor (I.F) by formula,

$$\boxed{\text{I.F} = e^{\int P(x)dx}}$$

2) General Solution is obtained by formula,

$$\boxed{y.(I.F)=\int Q(x).(I.F)dx+C}$$

For Example:- $(1+x^2)dy=(\tan^{-1}x-y)dx$

Solution :- Since equation contain only one term of y with first power. So equation is Linear.

Simplifying,

$$\left(1+x^2\right)\frac{dy}{dx} = \tan^{-1}x - y$$

Transferring' y' variable term on left side,

$$\left(1+x^2\right)\frac{dy}{dx} + y = x$$

Dividing $\left(1+x^2\right)$ on both side,

$$\frac{dy}{dx} + \left(\frac{1}{1+x^2}\right)y = \frac{x}{1+x^2}$$

Comparing with, $\boxed{\frac{dy}{dx} + P(x)y = Q(x)}$ $\quad P(x) = \frac{1}{1+x^2}$ & $Q(x) = \frac{x}{1+x^2}$

$$\boxed{\text{I.F.} = e^{\int \frac{1}{1+x^2}dx} = e^{\tan^{-1}x}}$$

Gen solution is,

$$y\left(e^{\tan^{-1}x}\right) = \int\left(\frac{x}{1+x^2}\right)e^{\tan^{-1}x}dx + C \quad \text{----------} \ (1)$$

Put $\tan^{-1}x = t \ \Rightarrow \ x = \tan t$

Diff. $\frac{1}{1+x^2}dx = dt$

Using substitution on right side of (1), it become

$$y\left(e^{\tan^{-1}x}\right) = \int e^t \tan t dt + C$$

By solving integration on right side & using back substitution,

Gen. Solution is $\boxed{ye^{\tan^{-1}x} = e^{\tan^{-1}x}\left(\tan^{-1}x - 1\right) + C}$

Assignment-6

1) Solve:- $\frac{dy}{dx} + y\tan x = \cos^2 x$

Solution:- This is linear equation in 'y' already expressed in the form $\boxed{\frac{dy}{dx} + P(x)y = Q(x)}$

Here $P(x) = \tan x$ & $Q(x) = \cos^2 x$

I. F. = $e^{\int \tan x dx} = e^{\log(\sec x)} = \sec x$ (Using log property, $e^{\log a} = a$)

$$\boxed{\therefore I.F = \sec x}$$

Gen. Solution is, $y\sec x = \int \cos^2 x \sec x dx + C$

Gen solution is --

2) Solve:- $x^2 \frac{dy}{dx} = 3x^2 - 2xy + 1$

<u>Solution</u>:- Transferring' y' variable term on left side,

$x^2 \frac{dy}{dx} + 2xy = 3x^2 + 1$ ------------ linear in y

Dividing by 'x^2' on both side

$\frac{dy}{dx} + \frac{2y}{x} = 3 + \frac{1}{x^2}$

Here, $P(x) = \frac{2}{x}$ & $Q(x) = 3 + \frac{1}{x^2}$

I. F. = $e^{\int \frac{2}{x} dx} = e^{2\int \frac{1}{x} dx} = e^{2\log x} = e^{\log x^2} = x^2$ ------Integrating factor

Gen. Solution is, $yx^2 = \int \left(3 + \frac{1}{x^2}\right) x^2 dx + C$

Gen solution is --

3) Solve:- $\frac{dy}{dx}+(1+2x)y=e^{-x^2}$

Solution:- This is linear equation in 'y' already expressed in the form $\boxed{\frac{dy}{dx}+P(x)y=Q(x)}$

Here, $P(x)=1+2x$ & $Q(x)=e^{-x^2}$

I.F. = $e^{\int(1+2x)dx}$ = e^{x+x^2} ------------------ Integrating factor

Gen solution is ---

4) Solve:- $\cos x\frac{dy}{dx}+y=\sin x$

Solution:- Dividing by 'cosx' on both side

Equation become,

$$\frac{dy}{dx}+y\sec x=\tan x$$

Here, $P(x)=\sec x$ & $Q(x)=\tan x$

I.F.=---

Gen solution is ---

Exercise:-6

Solve:- 1) $x\frac{dy}{dx}+y=x^3$ 2) $x\frac{dy}{dx}-y=x^2$ 3) $\frac{dy}{dx}++y\cot x=\cos ecx$

4) $\cos x\frac{dy}{dx}+2y\sin x=\sin 2x$ 5) $\left(x^2+1\right)\frac{dy}{dx}+2xy=\frac{1}{x^2+1}$ 6) $\cos^2 x\frac{dy}{dx}+y=\tan x$

5) Bernoulli's Diff. Equation :- The differential equation of the form $\boxed{\frac{dy}{dx}+P(x)y=Q(x)y^n}$ is called as '**Bernoulli's Diff. Equation**' where P(x) & Q(x) are function of 'x'or Constant.

Method of Solution:- Let $\boxed{\frac{dy}{dx}+P(x)y=Q(x)y^n}$ ------------------Bernoulli's equation

Divide both side by 'y^n'

$y^{-n}\frac{dy}{dx}+P(x)y^{1-n}=Q(x)$ -----------(1)

Put $\boxed{y^{1-n}=t}$ $\Rightarrow$ $\boxed{(1-n)y^{-n}\frac{dy}{dx}=\frac{dt}{dx}}$ or $\boxed{y^{-n}\frac{dy}{dx}=\frac{1}{1-n}\frac{dt}{dx}}$

Using Substitution,
From (1),

$$\frac{1}{1-n}\frac{dt}{dx}+P(x)t=Q(x)$$

Or $\frac{dt}{dx}+(1-n)P(x)t=(1-n)Q(x)$ ---------------Linear Diff. Equation in 't'

This diff. equation is further solved by Linear Diff. Equation Method

For Example:- Solve:- $\frac{dy}{dx}-xy=-y^3e^{-x^2}$

Solution: - It is Bernoulli's Diff. Equation

Dividing by ' y^3 ' on both side

$$y^{-3}\frac{dy}{dx}-xy^{-2}=-e^{-x^2} \text{ --------------------- (1)}$$

Put $\boxed{y^{-2}=t}$ $\Rightarrow$ $\boxed{-2y^{-3}\frac{dy}{dx}=\frac{dt}{dx}}$

Equation (1) become, $-\frac{1}{2}\frac{dt}{dx}-xt=-e^{-x^2}$

$$\frac{dt}{dx}+(2x)t=2e^{-x^2} \text{ ------------Linear equation}$$

$$P(x)=2x \ \& \ Q(x)=2e^{-x^2}$$

$$\text{I.F.}=e^{\int 2x dx}=e^{x^2}$$

Solution is, $te^{x^2}=\int 2e^{-x^2}e^{x^2}dx+C = 2\int dx+C$

$te^{x^2}=2x+C$ Using back substitution of 't'

Or, $\boxed{\frac{e^{x^2}}{y^2}=2x+C}$

Assignment-7

1) Solve, $\frac{dy}{dx}+\frac{y}{x}=y^3$

Solution: - It is Bernoulli's Diff. Equation

Dividing by ' y^3 ' on both side

$$\frac{1}{y^3}\frac{dy}{dx}+\left(\frac{1}{x}\right)\frac{1}{y^2}=1 \text{ ------------ (1)}$$

Put $\boxed{\frac{1}{y^2}=t}$ $\Rightarrow$ $\boxed{-\frac{2}{y^3}\frac{dy}{dx}=\frac{dt}{dx}}$ or $\boxed{\frac{1}{y^3}\frac{dy}{dx}=-\frac{1}{2}\frac{dt}{dx}}$

Equation (1) become,

$$-\frac{1}{2}\frac{dt}{dx}+\left(\frac{1}{x}\right)t=1$$

Or, $$\frac{dt}{dx}+\left(\frac{-2}{x}\right)t=-2 \text{ -----------------Linear equation}$$

Gen solution is ---

2) Solve :- $x\frac{dy}{dx}+y=x^3y^6$

Solution: - It is Bernoulli's Diff. Equation

By simplifying, $\frac{dy}{dx}+\frac{y}{x}=x^2y^6$

Divide by ' y^6 ' on both side

$$\frac{1}{y^6}\frac{dy}{dx}+\left(\frac{1}{x}\right)\frac{1}{y^5}=x^2$$

Gen solution is ---

3) Solve :- $x\frac{dy}{dx}+y=y^2\log x$

Solution:-

Gen solution is ---

Exercise:-7

Solve:-1) $\frac{dy}{dx}+xy=x^3y^3$ 2) $2y-3x\frac{dy}{dx}=e^xy^4$ 3) $x\frac{dy}{dx}-y=y^3\log x$

4) $\cos x\frac{dy}{dx}+y\sin x=y^4\sin 2x$ 5) $\frac{dz}{dx}+\frac{z}{x}\log z=\frac{z}{x^2}(\log z)^2$ (Hint: - First divide by 'z' then put logz = y)